Sain of problem

We have $\Omega AB \leq 1$ & so need $\Omega = 1/AB$

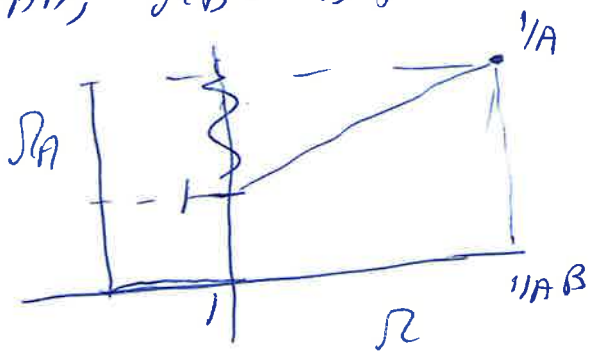
Write $\Omega AB = \Omega_A A \Omega_B B$.

where Ω_A, Ω_B so that Ω_B, Ω_A are monotone increasing

function of Ω on extend $(1, 1/AB)$ such

that $\Omega_A = 1/A$ for $\Omega = 1/AB$, $\Omega_B = 1/B$ for

$\Omega = 1/AB$



Try $\Omega_A = \alpha + \beta \Omega$.

Then $\dfrac{\Omega_A - 1}{\Omega - 1} = \dfrac{\frac{1}{A} - 1}{\frac{1}{AB} - 1}$

So $\Omega_A = 1 + \left(\dfrac{1-A}{1-AB}\right) B (\Omega - 1)$

$= 1 + B(1-A)\dfrac{B-AB}{1-AB} + \dfrac{B-AB}{1-AB} \cdot \Omega$.

$= \dfrac{1-AB-B+AB}{1-AB} + \dfrac{B-AB}{1-AB} \cdot \Omega$

$= \dfrac{1-B}{1-AB} + \dfrac{B-AB}{1-AB} \Omega$.

$\alpha = \dfrac{1-B}{1-AB}$ $\quad \beta = \dfrac{B-AB}{1-AB}$

$\alpha, \beta > 0$. $\quad \alpha + \beta = 1$

and $\Lambda_B = \frac{\Omega}{\Omega_A} = \frac{\Omega}{\alpha + \beta\Omega}$.

$\Omega = 1, \Lambda_B = 1,$

$\Omega = \frac{1}{AB}, \Lambda_B = 1/B$.

$$\frac{d\Omega_A}{d\Omega} = \beta > 0$$

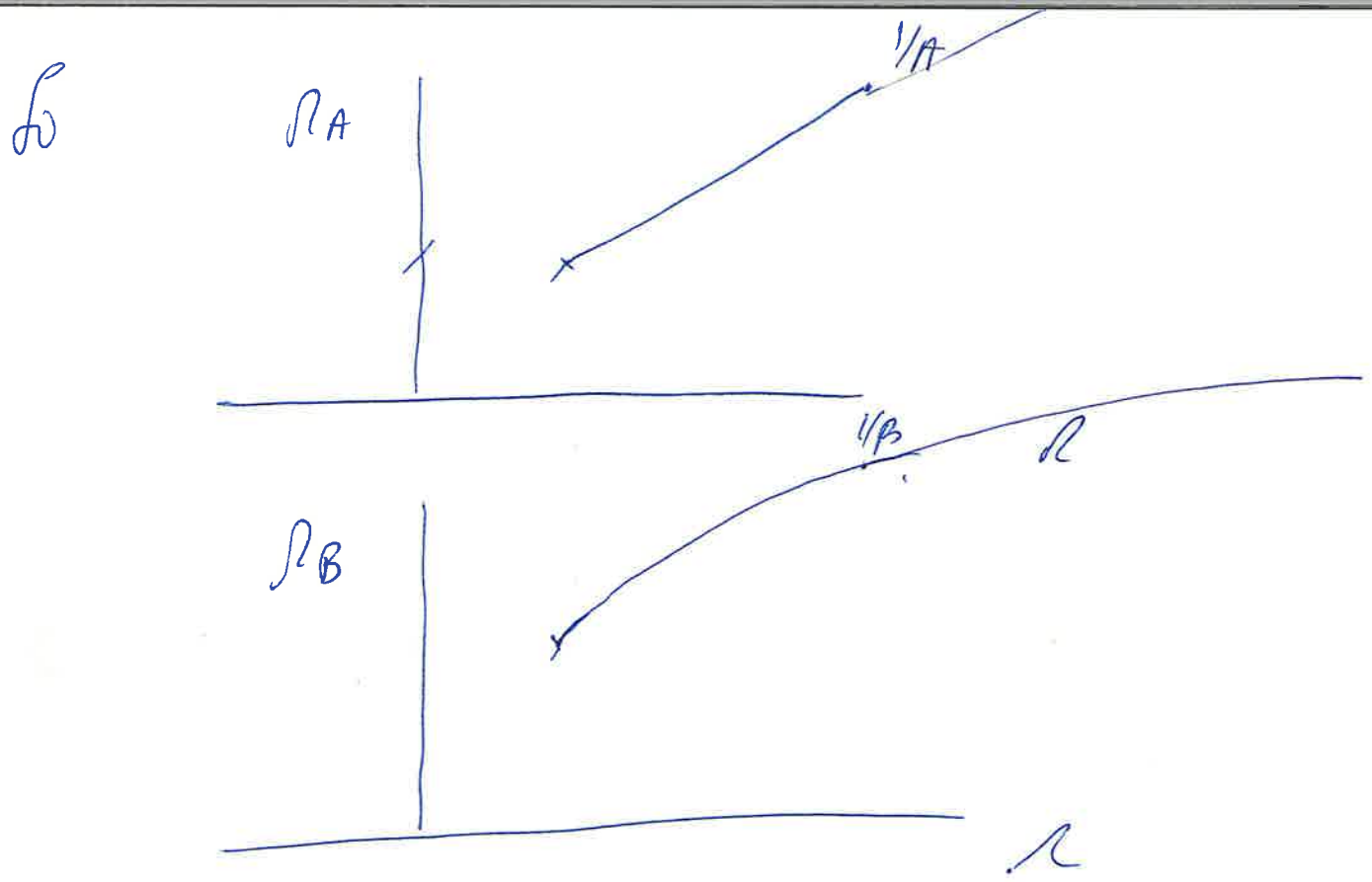
and $$\frac{d\Omega_B}{d\Omega} = \frac{(\alpha+\beta\Omega) - \Omega\beta}{(\alpha+\beta\Omega)^2} = \frac{\alpha}{(\alpha+\beta\Omega)^2} > 0.$$

In all conditions are satisfied.

$$\frac{d^2\Lambda_B}{d\Omega^2} = \frac{-2\alpha\beta}{(\alpha+\beta\Omega)^3}.$$ is rejected.

$\Omega \to \infty \quad \Lambda_B \to 1/\beta = \frac{1-AB}{B(1-A)} > \frac{1}{B}$

if $1-AB > 1-A$
or $AB < A$ or $B < 1$ which
true.



else if we calculate value of

$\Lambda_A^{(1)}$ from $\Lambda^{(1)}$ → $A^{(1)} = \Lambda_A^{(1)} \cdot A$.

$B^{(1)} = \Lambda_B^{(1)} B$.

Now use this value.

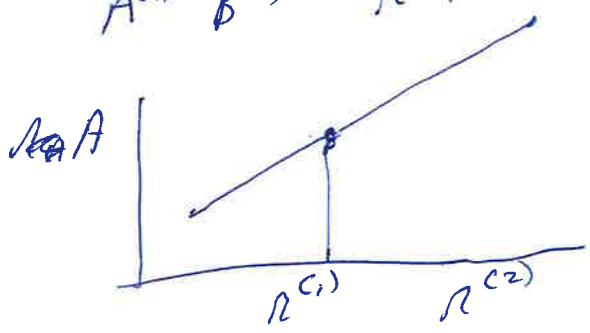
for $A^{(1)}$, $B^{(1)}$ to compute $A^{(2)}$ from $\Lambda^{(2)}$

$\Lambda^{(2)}$ for next value $\frac{1}{A^{(1)}} \frac{1}{B^{(1)}} = \frac{1}{\Lambda^{(1)} AB} =$

for $\Lambda^{(1)}$ $\Lambda^{(2)}$ less

next value $\frac{1}{AB}$

~~But $\Lambda^{(1)}$ $\Lambda^{(2)}$~~



In general $\Lambda = \Lambda^{(1)} \Lambda^{(2)}$

and use old construction for Λ.

$\Lambda^{(2)} = 1$ corresponds to $A = A^{(1)}$

and $\Omega^{(2)} = \frac{1}{A^{(1)} B^{(1)}}$ corresp. to

$$\Omega = \Omega^{(1)} \cdot \Omega^{(2)} = \Omega^{(1)} \frac{1}{A^{(1)} B^{(1)}} = \frac{\Omega^{(1)}}{\Omega^{(1)} A B} = \frac{1}{AB}$$

So using $\Omega^{(1)}$, $\Omega^{(2)}$ in succession gives same result as using $\Omega^{(1)} \Omega^{(2)}$ in one step.

we can define $\sqrt{\Omega} R(\Omega) = \Omega_A$

then $R(\Omega) = \frac{\Omega_A}{\sqrt{\Omega}}$.

and $\sqrt{\Omega} / R(\Omega) = \Omega_B$

so $R(\Omega) = \frac{\sqrt{\Omega}}{\Omega_B}$

then $R(\Omega) = \frac{\alpha + \beta \Omega}{\sqrt{\Omega}} =$

$$= \frac{1}{\sqrt{\Omega}} \left(\frac{1-B}{1-AB} + \frac{B-AB}{1-AB} \cdot \Omega \right)$$

$$R(\Omega) = \frac{1}{(1-AB)\sqrt{\Omega}} \left((1-B) + B(1-A)\Omega \right)$$

So in our main formula

we have $\sqrt{x} = B$,

$f_c'(x_1 \dots x_{n-1}) = A$

and we want $\textcircled{H}\left(\sqrt{x}, f_c'(x_1 \dots x_{n-1}), \ell^{(x_n)}\right)$

$\downarrow c$

$$\textcircled{H}(A,B,C) = \frac{1}{(1-AB)\sqrt{C}}\left((1-A) + A(1-B)C\right)$$

)

$A = \sqrt{x}, \quad B = f_c', \quad C = \ell^{(x_n)}$

thus our formula is correct.

with

$$\textcircled{H}(A,B,C) = \frac{1}{(1-AB)\sqrt{C}}\left(1 - A + A(1-B)C\right)$$

$$\underline{\Phi}(A,B,C) = \frac{1}{C(1-A^B) + A^B}$$

~~R ∈ S.~~ $\alpha = \alpha\alpha'' + \alpha''\beta\Omega$, $\beta = \beta\beta'' + \alpha\beta''/\Omega$

Consistency for Ω_A - Rule

Thus $\Omega_A = \alpha + \beta\Omega$. α or α ~~Type~~

$\Omega_A = \Omega_A' \Omega_A''$, $\Omega = \Omega' \Omega''$

where $\Omega_A' = \alpha' + \beta'\Omega'$
$\Omega_A'' = \alpha'' + \beta''\Omega''$

$\alpha = \frac{1-B}{1-AB}$, $\beta = \frac{B-AB}{1-AB}$

$\alpha' = \frac{1-B'}{1-A'B'}$, $\beta' = \frac{B'-A'B'}{1-A'B'}$

$$B' = \Omega_B' B = \frac{\Omega'}{\alpha+\beta\Omega'} \cdot B$$

$$A' = \Omega_A' A = (\alpha+\beta\Omega')A$$

we require $\alpha + \beta\Omega = (\alpha+\beta\Omega')(\alpha'+\beta'\Omega'')$

$$= (\alpha\alpha' + \alpha'\beta\Omega') + \frac{\alpha\beta'}{\Omega'} \cdot \Omega + \beta\beta'\Omega$$

& we require $\alpha = \alpha\alpha' + \alpha'\beta\Omega'$, $\beta = \beta\beta' + \frac{\alpha\beta'}{\Omega'}$

Now $T = \alpha\alpha' + \alpha'\beta\gamma'$

$\alpha' = \dfrac{1-B'}{1-A'B'} = \dfrac{1-\dfrac{\gamma'B}{\alpha+\beta\gamma'}}{1-\gamma'AB}$

$T = \dfrac{1-B}{1-AB} \cdot \dfrac{1-\dfrac{\gamma'B}{\alpha+\beta\gamma'}}{1-\gamma'AB} +$

$\alpha'(\alpha+\beta\gamma') = \alpha' \cdot \dfrac{A'}{A}$

$= \dfrac{\alpha+\beta\gamma'-\gamma'B}{1-\gamma'AB}$, now $\alpha+\beta\gamma'$

$= \dfrac{1-B}{1-AB} + \dfrac{B-AB}{1-AB}\cdot\gamma'$

$= \dfrac{(1-B)+B(1-A)\gamma'}{1-AB}$

& $T = \dfrac{\dfrac{1-B+B(1-A)\gamma'}{1-AB} - \gamma'B}{1-\gamma'AB}$

$= \dfrac{1-B+B(1-A)\gamma' - \gamma'B(1-AB)}{(1-AB)(1-\gamma'AB)}$

$= \dfrac{1-B-AB\gamma'+AB^2\gamma'}{(1-AB)(1-\gamma'AB)} = \dfrac{(1-B)(1-\gamma'AB)}{(1-AB)(1-\gamma'AB)}$

$= \dfrac{1-B}{1-AB} = \alpha$ as required

Now let $\bar{T} = \beta\beta' + \frac{\alpha\beta'}{\lambda'} = \beta'\left(\beta + \alpha/\lambda'\right)$

$$\beta' = \frac{B' - A'B'}{1 - A'B'} = \frac{\frac{\lambda'}{\alpha+\beta\lambda'} \cdot B - \lambda' AB}{1 - \lambda' AB}$$

and $\beta + \alpha/\lambda' = \frac{B(1-A)}{1-AB} + \frac{1-B}{\lambda'(1-AB)} = \frac{\lambda' B(1-A) + 1 - B}{\lambda'(1-AB)}$

$$\delta\bar{T} = \frac{\lambda' B - \lambda' AB(\alpha+\beta\lambda')}{(1-\lambda' AB)(\alpha+\beta\lambda')} \cdot \frac{(\alpha+\beta\lambda')}{\lambda'}$$

$$= \frac{B - AB(\alpha+\beta\lambda')}{1-\lambda' AB} = B\left(\frac{1 - A(\alpha+\beta\lambda')}{1-\lambda' AB}\right)$$

$$= \frac{\beta}{1-\lambda' AB}\left[1 - A\left(\frac{1-B+B(1-A)\lambda'}{1-AB}\right)\right]$$

$$= \frac{\beta}{1-\lambda' AB}\left[\frac{1 - A\bar{B} - \bar{A} + AB - AB(1-A)\lambda'}{1-AB}\right]$$

$$= \frac{\beta}{(1-AB)(1-\lambda' AB)}\left[1 - A - AB\lambda' + A^2 B\lambda'\right]$$

$$= \frac{B(1-A)(1-\lambda' AB)}{(1-AB)(1-\lambda' AB)} = \frac{B(1-A)}{1-AB} = \beta$$

as required

A simpler consistency proof is as follows

$\Lambda_A = \alpha + \beta \Lambda$.

Introduce $\Lambda_A'' = \Lambda_A / \Lambda_A'$ $\quad \Lambda'' = \Lambda / \Lambda'$

Then $\Lambda_A'' \cdot \Lambda_A' = \alpha + \beta \Lambda' \cdot \Lambda''$

or $\Lambda_A'' = \frac{\alpha}{\Lambda_A'} + \frac{\beta \Lambda'}{\Lambda_A'} \cdot \Lambda''$. ——— (1)

where $\Lambda_A' = \alpha + \beta \Lambda'$.

We require to prove that (1) passes through the points $\begin{cases} \Lambda_A'' = 1 \text{ for } \Lambda'' = 1 \\ \Lambda_A'' = \frac{1}{A'} \text{ for } \Lambda'' = \frac{1}{A'B'} \end{cases}$ where $A' = \Lambda_A' A$, $B' = \Lambda'/\Lambda_A' \cdot B \left(= \frac{1}{\Lambda_B'} B\right)$

i.e. we require that the two families of our Λ_A curves

Put $\Lambda'' = 1$, $\Lambda_A'' = \frac{\alpha + \beta \Lambda'}{\Lambda_A'} = \frac{\Lambda_A'}{\Lambda_A'} = 1$ //

Put $\Lambda'' = \frac{1}{A'B'}$, $\Lambda_A'' = \frac{\alpha}{\Lambda_A'} + \frac{\beta \Lambda'}{\Lambda_A'} \cdot \frac{1}{A'B'}$

$= \frac{\alpha}{\Lambda_A'} + \frac{\beta}{\Lambda_A'} \frac{\Lambda'}{\Lambda' AB} = \frac{\alpha}{\Lambda_A'} + \frac{\beta}{\Lambda_A' \cdot AB}$

$= \frac{1}{\Lambda_A'} \left(\alpha + \frac{\beta}{AB}\right) = \frac{1}{\Lambda_A'} \cdot \frac{1}{A}$

given $\alpha + \beta \cdot \frac{1}{AB} = \frac{1}{A}$
for some α, β.

$= \frac{A}{A'} \cdot \frac{1}{A} = \frac{1}{A'}$ //

But notice as $A = B$

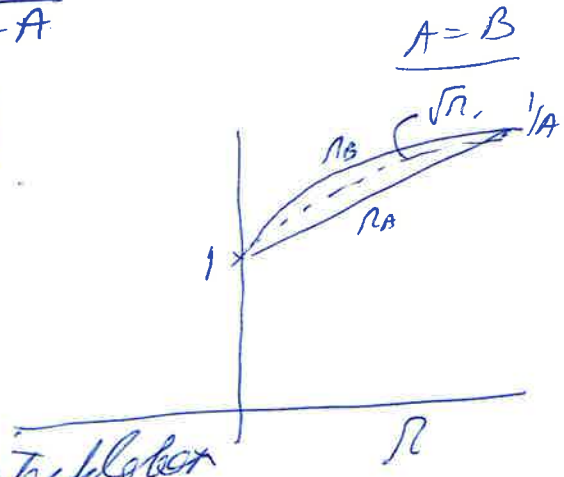
we get

$$\Lambda_A = \frac{1-A}{1-A^2} + \frac{A(1-A)}{1-A^2}\cdot r$$

$$= \frac{1}{1+A} + \frac{Ar}{1+A} = \frac{1+Ar}{1+A}$$

$$\Lambda_B = \frac{r}{\Lambda_A} = \frac{r(1+A)}{1+Ar}$$

& $\Lambda_A \neq \Lambda_B$



Λ_A approximates $\sqrt{r}$ by linear interpolation through fixed end-points at $r=1$ and $r = 1/A^2$.

Need to show $\Lambda_A - \Lambda_B = \frac{1+Ar}{1+A} - \frac{r(1+A)}{1+Ar} = \frac{(1-r)(1-rA^2)}{(1+A)(1+Ar)}$ //

< 0 since $r > 1$

and $\Lambda_A - \sqrt{r} = \frac{1+Ar}{1+A} - \sqrt{r} = \frac{1+Ar-\sqrt{r}-\sqrt{r}A}{1+A}$

$$= \frac{(1-\sqrt{r})(1-A\sqrt{r})}{1+A} \quad // \; < 0$$

and $\Lambda_B - \sqrt{r} = \frac{r(1+A)}{1+Ar} - \sqrt{r} = \frac{r+Ar-\sqrt{r}-Ar^{3/2}}{1+Ar}$

$$= \sqrt{r}\left(\sqrt{r}-1\right)\frac{(1-A\sqrt{r})}{1+Ar} \quad > 0 \quad //$$

($\sqrt{r} > 1$)

In the choice $\Lambda_A = \alpha + \beta\, n \quad \alpha \quad \frac{n}{\alpha + \beta n}$

intersects $\Lambda_A \propto 1/A$ at consistency condition but does not reduce to $\Lambda_A = \sqrt{n}$ for $A = B$

this last condition can not probably not be satisfied if first two conditions are satisfied. (?)

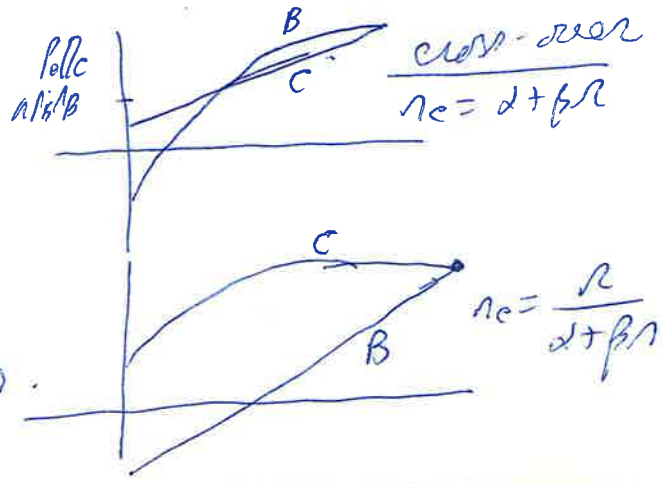
Choice $\alpha + \beta n \quad \alpha \quad \frac{n}{\alpha + \beta n}$ for Λ_C

is intended, either goes on adequate model for applying a figure of merit to Λ_C in accordance another methodological rule. Choose $\Lambda_C = \alpha + \beta n$ be called etc as the simplest choice (Occam interpretation of $\sqrt{n}$). But choice $\Lambda_C = \frac{n}{\alpha + \beta n}$ makes

$\Lambda_C > \Lambda_B$ if $P(C) > P(B)$ this means we always favour the hard-core at the expense of the protective belt though both share an the enhancement

Λ_C $+P(C)$

thus



cross-over $\Lambda_C = \alpha + \beta n$

$\Lambda_C = \frac{n}{\alpha + \beta n}$.

we want no cross-over.

General consistency equation

$\Omega_A'' = f(\Omega)$ — (1) f of a certain form

Substitute $\Omega_A'' = \Omega_A / \Omega_{A'}$ $\quad \Omega'' = \Omega / \Omega'$

So $\Omega_A'' \cdot \Omega_{A'} = f(\Omega' \Omega'')$ — (2)

Now for $\Omega'' = 1$ $\quad \Omega_A'' = 1$ $\quad$ hence $\Omega_{A'} = f(\Omega')$

" " $\Omega'' = \frac{1}{A'B'}$ $\quad A'B' = \Omega' AB$

$\Omega_A'' \Omega_{A'} = f\left(\Omega' / A'B'\right) = f\left(\frac{1}{AB}\right) = \frac{1}{A}$

$\therefore \Omega_A'' = \frac{1}{A \Omega_{B'}} = \frac{1}{A'}$.

$\therefore \Omega_A'' = \frac{1}{\Omega_{A'}} f(\Omega', \Omega'')$ satisfies all our conditions

and must be equal to $f(\Omega'')$

So we require $\frac{1}{\Omega_{A'}} f(\Omega' \Omega'') = f(\Omega'')$

or $f(x)$ must have the property

$m f(nx)$ is of same form as $f(x)$

Eg $f(x) = \alpha x + \beta$, $\quad m f(nx) = \alpha m n \cdot x + \beta = \alpha' x + \beta$

So condition is satisfied

while $f(x) = x + \beta$ the 3rd condition is not satisfied.

Choices

(a) $\Omega_A = \sqrt{\Omega}\,(\alpha + \beta\Omega)$ $\quad \Omega_A \cdot A \cdot \cos 2\omega > 1$

(b) $\Omega_A = \alpha + \beta\Omega$. $\quad \Omega_A \neq \sqrt{\Omega}$ for $A = B$

(c) $\Omega_A = \sqrt{\Omega} + \alpha(1-\Omega)$ breaks consistency obstacles.

3 conditions to be satisfied

① $\Omega_A A \geq 1$

② consistency

③ $\Omega_A \to \sqrt{\Omega}$ for $A = B$.

also

④ $\Omega_A A > \Omega_B B$

iff $A > B$.

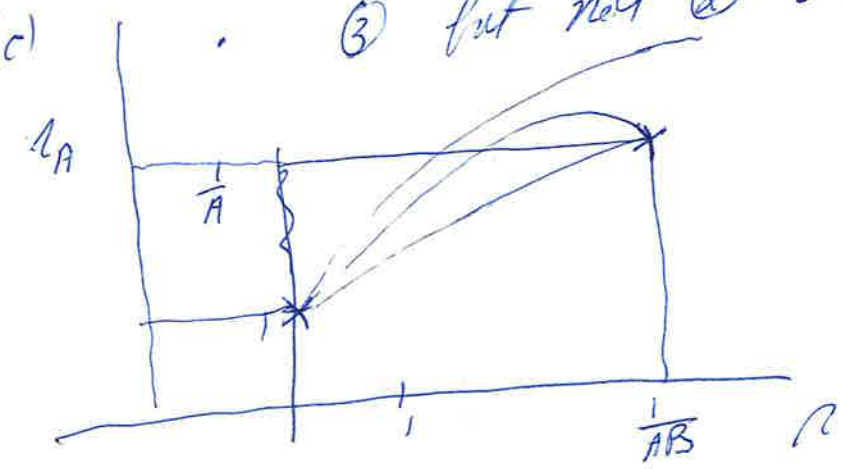
↓

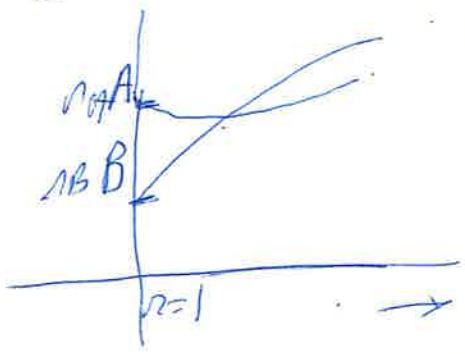
satisfied by $\Omega_A = \dfrac{\Omega}{\alpha + \beta\Omega}$.

(d) satisfies ② & ③ but not ①

(e) " ① & ② but not ③

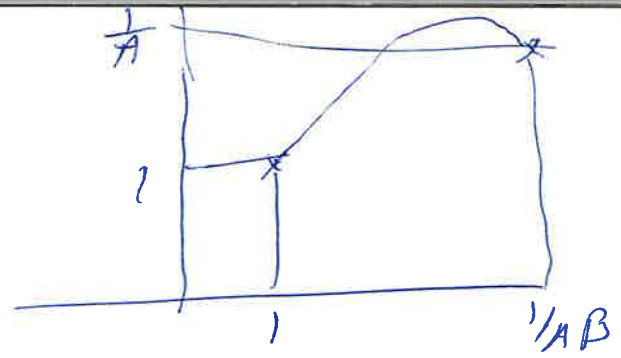
(f) " ③ but not ② or ①





Consider

$$\Lambda_A = \sqrt{\Lambda} - (\alpha + \beta \Lambda)$$



$\Lambda = 1$, $\Lambda_A = 1$

pour $\alpha + \beta = 0$.

$$\Lambda_A = \sqrt{\Lambda} + \alpha(1 - \Lambda)$$

$\Lambda = 1/AB$, $\Lambda_A = 1/A$ $\Rightarrow$

$$\frac{1}{A} = \frac{1}{\sqrt{AB}} + \alpha\left(1 - \frac{1}{AB}\right)$$

$$\alpha = \frac{\frac{1}{A} - \frac{1}{\sqrt{AB}}}{1 - \frac{1}{AB}} = \frac{AB}{AB-1} \cdot \frac{\sqrt{AB} - A}{A\sqrt{AB}}$$

$$= \frac{B}{AB-1} \cdot \frac{\sqrt{AB} - A}{\sqrt{AB}} = \frac{B}{1-AB} \cdot \frac{A\left(1 - \sqrt{B/A}\right)}{\sqrt{AB}}$$

$A = B$, $\alpha = 0$

$A > B$, $\alpha > 0$

Par $\frac{d\Lambda_A}{d\Lambda} = \frac{1}{2\Lambda^{1/2}} \Rightarrow \alpha$ toujours > 0 si

$$\frac{1}{2\sqrt{AB}} > \frac{B}{1-AB} \frac{A\left(1 - \sqrt{B/A}\right)}{\sqrt{AB}}$$

ou $1 - AB > 2AB\left(1 - \sqrt{B/A}\right)$

$$\frac{1}{2} > \frac{AB}{1-AB}\left(1 - \sqrt{B/A}\right)$$

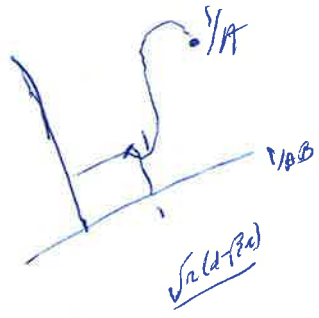
ou $1 > 3AB - 2B^{3/2}A^{1/2}$

$AB = \varepsilon$, $B^{3/2}A^{1/2} = \eta$ $\quad 1 > 3\varepsilon - 2\eta = 3AB\left(1 - \frac{2}{3}\sqrt{\frac{B}{A}}\right)$

$\therefore$

$$\left.\frac{d\eta_A}{dR}\right|_{R=1} = \frac{1}{2} - \beta$$

$$< 0 \quad \text{if} \quad \frac{1}{2} < \beta$$



or $\frac{1}{2} > \beta$ (i.e. $\frac{1}{2} > \frac{1-\sqrt{B/A}}{1/AB - 1}$

$$\frac{1}{AB} - 1 > 2 - 2\sqrt{B/A}$$

$$1 - AB > 2AB - 2\sqrt{A}B^{3/2}$$

$$1 > 3AB - 2\sqrt{A}B^{3/2}$$

$$= 3\varepsilon - 2\eta \qquad \varepsilon < 1,\ \eta < 1$$

or $1 - AB > 2AB(1 - \sqrt{B/A})$

Try $\Omega_A = \sqrt{n}\, R(n)$

$$\frac{d\Omega_A}{dn} = \frac{1}{2\sqrt{n}} R(n) + \sqrt{n}\, R'(n)$$

$$R(n) = \alpha - \beta n \qquad R(1) = 1 \qquad R\left(\frac{1}{AB}\right) = \sqrt{B/A}$$

$$\beta = \frac{1 - \sqrt{B/A}}{\frac{1}{AB} - 1}$$

$$\frac{d\Omega_A}{dn} = \frac{1}{2\sqrt{n}} (\alpha - \beta n) - \sqrt{n}\,\beta \qquad \beta > 0$$

$$= \frac{\alpha}{2\sqrt{n}} - \frac{\beta\sqrt{n}}{2} - \beta\sqrt{n}$$

For zero $n = 1/AB$.

$$\frac{d\Omega_A}{dn} = \frac{\sqrt{AB}}{2} \cdot \sqrt{B/A} - \frac{1}{\sqrt{AB}} \cdot \frac{1 - \sqrt{B/A}}{\frac{1}{AB} - 1}$$

> 0 if $B < A$.

$$\frac{\beta}{2} > \sqrt{AB} \cdot \frac{1 - \sqrt{B/A}}{1 - AB}$$

$$\sqrt{B} > 2\left(\frac{1 - \sqrt{B/A}}{1 - AB}\right) \cdot \sqrt{A}$$

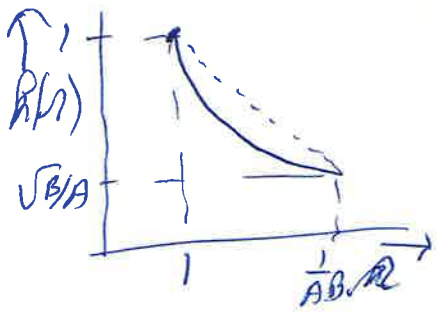
For small B this will be false as L.H.S. $\to 0$, R.H.S. $\to 2\sqrt{A}$.

at $n = 1$ $\frac{d\Omega_A}{dn} = \frac{1}{2} - \beta$

Now try $R_A = \sqrt{R(\lambda)}\,(\alpha - \beta\lambda)^2$. $R(\lambda) = (\alpha-\beta\lambda)^2$

Then $\frac{dI_A}{d\lambda} = \frac{1}{2\sqrt{\lambda}}(\alpha-\beta\lambda)^2 - \sqrt{\lambda}\,\beta\,2(\alpha-\beta\lambda)$

Given $R(1) = 1$, $\alpha - \beta = 1$



$R\left(\frac{1}{AB}\right) = \sqrt{B/A}$

$\left(\alpha - \beta/AB\right)^2 = \sqrt{B/A}$.

$\left(1 + \beta - \beta/AB\right)^2 = \sqrt{B/A}$.

$\therefore \left(1 - \beta\left(\frac{1}{AB} - 1\right)\right)^2 = \sqrt{B/A}$.

$\therefore \beta = \frac{1 - \sqrt[4]{B/A}}{\frac{1}{AB} - 1}$.

at $\lambda = 1/AB$

$\frac{dI_A}{d\lambda} = \frac{\sqrt{AB}}{2\sqrt{AB}}\cdot\sqrt{B/A} - \frac{1}{\sqrt{AB}}\cdot 2\cdot\frac{1-\sqrt[4]{B/A}}{\frac{1}{AB}-1}\cdot\sqrt[4]{B/A}$

$= \frac{B}{2} - \sqrt{AB}\cdot 2\cdot\frac{1-\sqrt[4]{B/A}}{1-AB}\cdot\sqrt[4]{B/A}$.

≥ 0 if

$\sqrt{B} \geq 4\,\frac{1-\sqrt[4]{B/A}}{1-AB}\cdot\sqrt{A}\cdot\sqrt[4]{B/A}$

or $\sqrt[4]{B} \geq 4\cdot\frac{1-\sqrt[4]{B/A}}{1-AB}\cdot\sqrt[4]{A}$. ✱ check again scale for small λ.

$f(\Omega')$

$f(\Omega'\Omega'')$ has same form as $f(\Omega'')\,f(\Omega')$

$= \lambda + \beta' B' \Omega'$

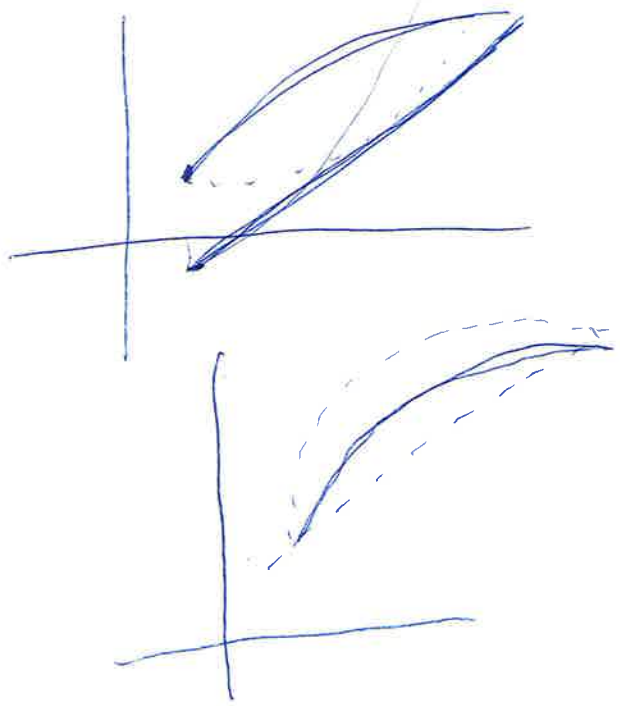
$f(A, B, \Omega'\Omega'') = f(A, B, \Omega') \times f\left[F(A,B,\Omega')A,\ \frac{\Omega}{F(A,B,\Omega')}B,\ \Omega'' \right]$

$f(A, B, \Omega'\Omega'') = f(A, B, \Omega') \times f\left[f(A,B,\Omega')A,\ \frac{\Omega'}{f(A,B,\Omega')} \cdot B,\ \Omega'' \right]$

Einstein Gyldelöw Selected by

$$f(A,B,\Omega) = \frac{1-B+B(1-A)\Omega}{1-AB}$$

$$\Omega = \frac{\Omega(1-AB)}{1-B+B(1-A)\Omega}$$



$\alpha + \beta\Omega$

$\alpha \dfrac{\Omega}{\alpha + \beta\Omega}$

$\dfrac{\Omega A}{\alpha + \beta\Omega} > B(\alpha + \beta\Omega)$ $\quad A > B$

$\dfrac{\Omega A}{\beta} > (\alpha + \beta\Omega)^2 = \alpha^2 + \beta^2\Omega^2 + 2\alpha\beta\Omega.$

$\left(\frac{1}{\omega}\right)^0 = 1$

$\frac{1}{}$

$$= \frac{\ln\beta}{2\sqrt{n}\ln^2\beta n}\left[\ln\beta + \ln n - 2\right]$$

$= 0$ when $\beta n = e^2$ or $n = 2/\beta$

$$n_A = \sqrt{n}\, e^{-\alpha(n-1)}$$

$$\frac{1}{A} = \sqrt{\frac{1}{AB}}\, e^{-\alpha\left(\frac{1}{AB}-1\right)}$$

$$\sqrt{B/A} = e^{-\alpha\left(\frac{1}{AB}-1\right)}$$

$$\frac{1}{2}\ln A/B = \alpha\left(\frac{1}{AB}-1\right)$$

$$\alpha 2 = \frac{1}{2}\frac{\ln A/B}{\frac{1}{AB}-1}$$

$$\frac{dn_A}{dn} = \frac{1}{2\sqrt{n}}e^{-\alpha(n-1)} - \sqrt{n}\cdot\alpha\, e^{-\alpha(n-1)}$$

$$= \frac{e^{-\alpha(n-1)}}{2\sqrt{n}}\left(1 - 2n\alpha\right)$$

$$1 - 2n\alpha = 1 - \frac{n\ln A/B}{\frac{1}{AB}-1} > 0$$

$n = 1$ always
$\ln A/B < \frac{1}{AB} - 1$.

if $\frac{1}{AB} - 1 - n\ln A/B > 0$

or $n < \dfrac{\frac{1}{AB}-1}{\ln A/B}$.

$n = \frac{1}{AB}$ we need $\frac{1}{AB} < \dfrac{\frac{1}{AB}-1}{\ln A/B}$

or $\dfrac{\ln A/B}{AB} < \frac{1}{AB} - 1$

$\ln A/B < 1 - AB$
useful if small B.

$$\Omega_A = \frac{\alpha\sqrt{\Omega}}{\ln\beta\Omega} \qquad \Omega_A(1) = 1,\ \Omega_A\left(\frac{1}{AB}\right) = 1/A$$

$$\therefore \alpha = \ln\beta \qquad \frac{\ln\beta \cdot \sqrt{\Omega}}{\ln\beta\Omega}$$

$$\text{and} \quad \frac{\ln\beta \cdot \sqrt{\frac{1}{AB}}}{\ln \beta/AB} = \frac{1}{A}.$$

$$\therefore \sqrt{A/B}\ln\beta = \ln\beta + \ln\frac{1}{AB}$$

$$\therefore \ln\beta\left(\sqrt{A/B} - 1\right) = \ln\frac{1}{AB}$$

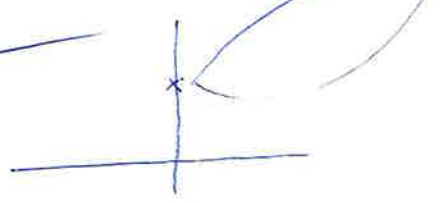
$$\therefore \ln\beta = \frac{1}{\sqrt{A/B} - 1} \cdot \ln\frac{1}{AB}$$

$$\alpha \quad \beta = \left(\frac{1}{AB}\right)^{\left(\frac{1}{\sqrt{A/B} - 1}\right)}$$

$$\frac{d\Omega_A}{d\Omega} = \frac{\ln\beta\Omega \times \ln\beta \cdot \frac{1}{2\sqrt{\Omega}} - \ln\beta \cdot \sqrt{\Omega} \cdot \beta/\beta\Omega}{\ln^2\beta\Omega}$$

$$= \frac{\ln\beta}{2\sqrt{\Omega}\ln^2\beta\Omega}\left[\ln\beta\Omega - 2\right]$$

$$\Rightarrow \text{if} = \frac{\ln^2\beta}{2\sqrt{\Omega}\ln^2\beta\Omega}\left[\ln\Omega - \ln\beta\right]$$

$\to 0$ when $\ln\Omega > 2$ or $\Omega > e^2 \approx 9$

$$= \frac{\ln\beta}{2\sqrt{\Omega}\ln^2\beta\Omega}\ln\Omega/\beta$$